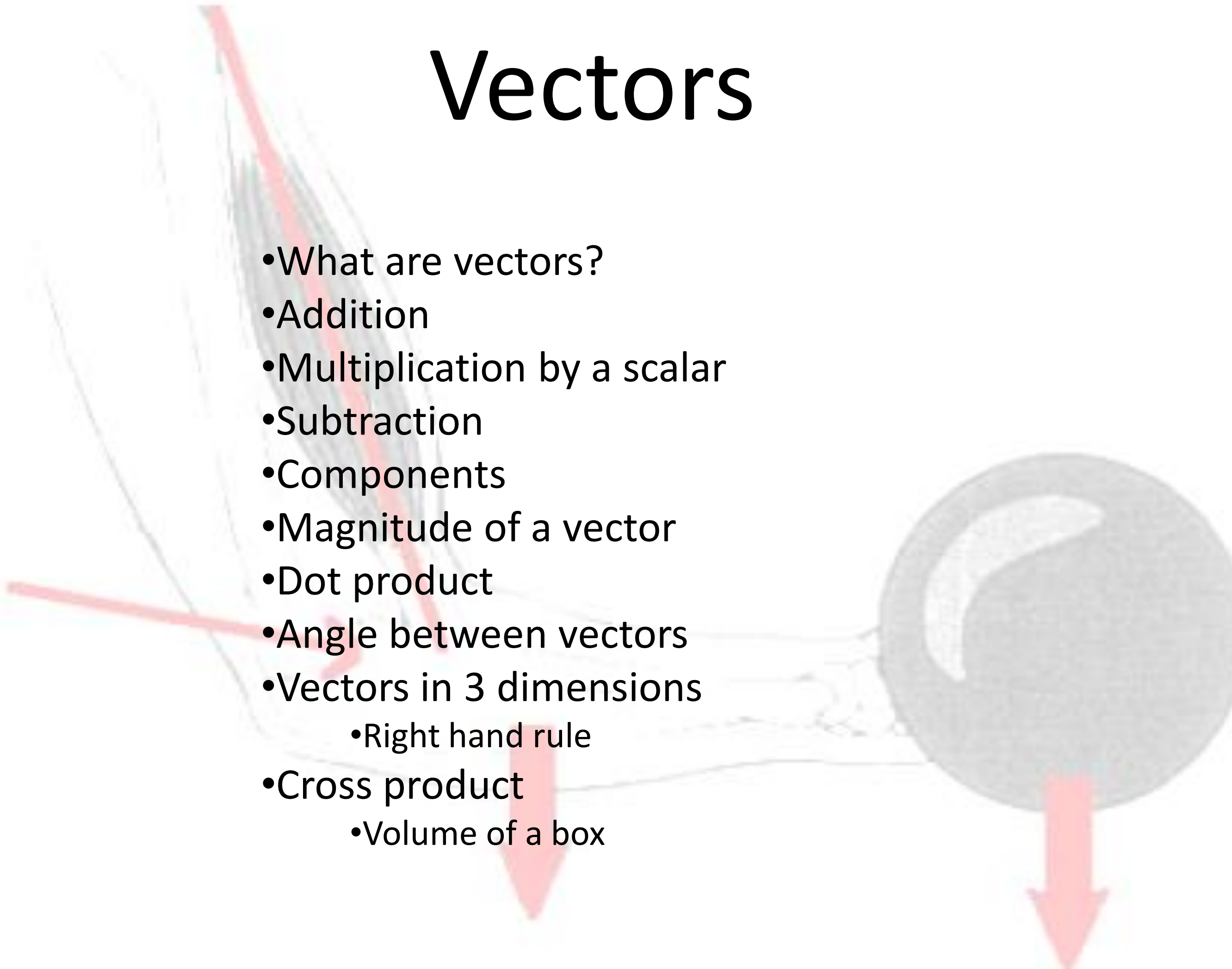
Vectors



- What are vectors?
- Addition
- Multiplication by a scalar
- Subtraction
- Components
- Magnitude of a vector
- Dot product
- Angle between vectors
- Vectors in 3 dimensions
    - Right hand rule
- Cross product
    - Volume of a box

What are Vectors?

- Lots of quantities we've met so far just have a size, and that's all
- Example; energy. Just need to know how much there is, no other info required
- But vectors also have direction
  - Need to specify where they're going
- Seen this already with quantities like velocity
  - Velocity can be positive or negative along one dimension
  - Depends whether it goes forwards or backwards

What are Vectors?

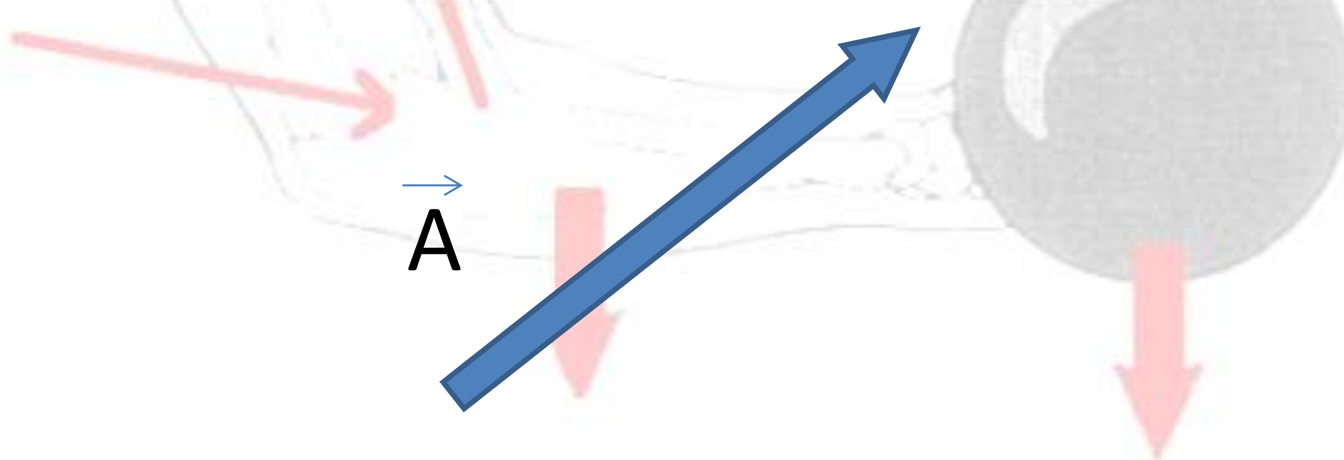
- Having a sign (+ve or –ve) isn't enough
    - Temperature, for example, isn't a vector
- The sign has to convey the concept of direction

Representing Vectors

- When written in text, we'll use an arrow over the vector name to indicate that this is a vector
  - Examples: $\vec{F}, \vec{v}, \vec{p}, \vec{L}, \vec{\omega}$
- Textbooks often use bold type to indicate vectors (e.g. **F, v, p, L, ω**)

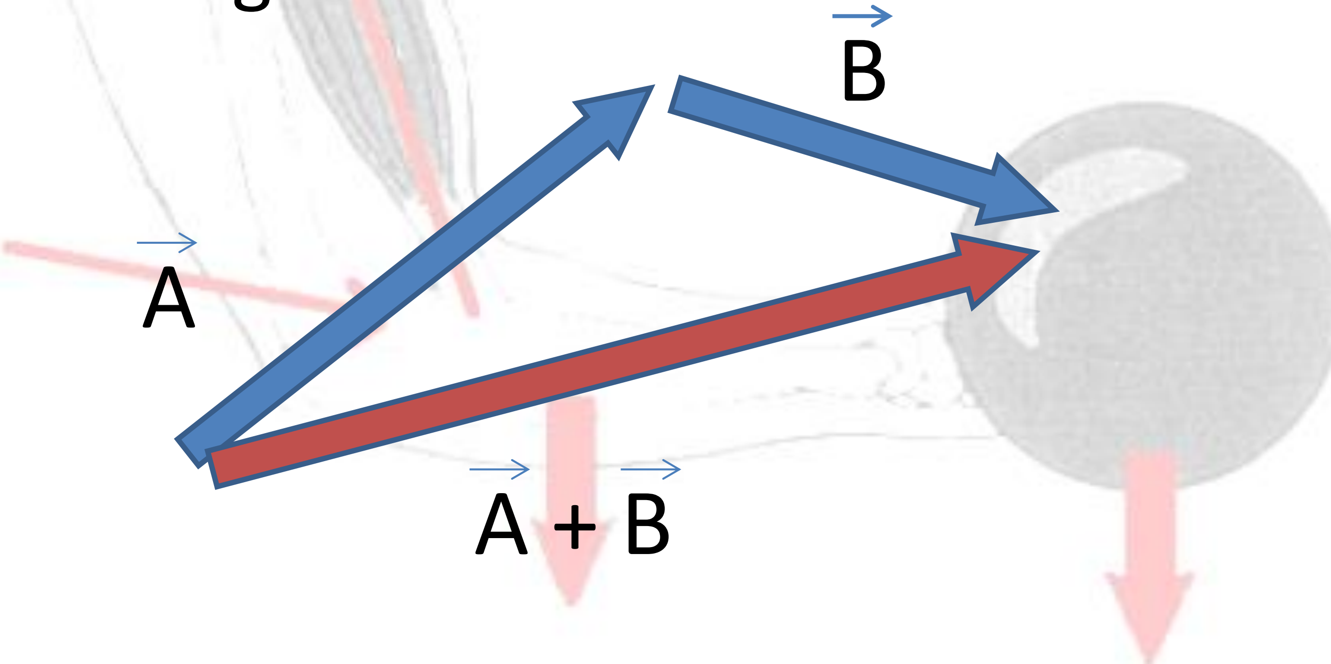
Representing Vectors

- Graphically, we'll draw vectors as arrows
  - the length indicates the size of the vector
  - the direction of the arrow is the same as that of the vector.



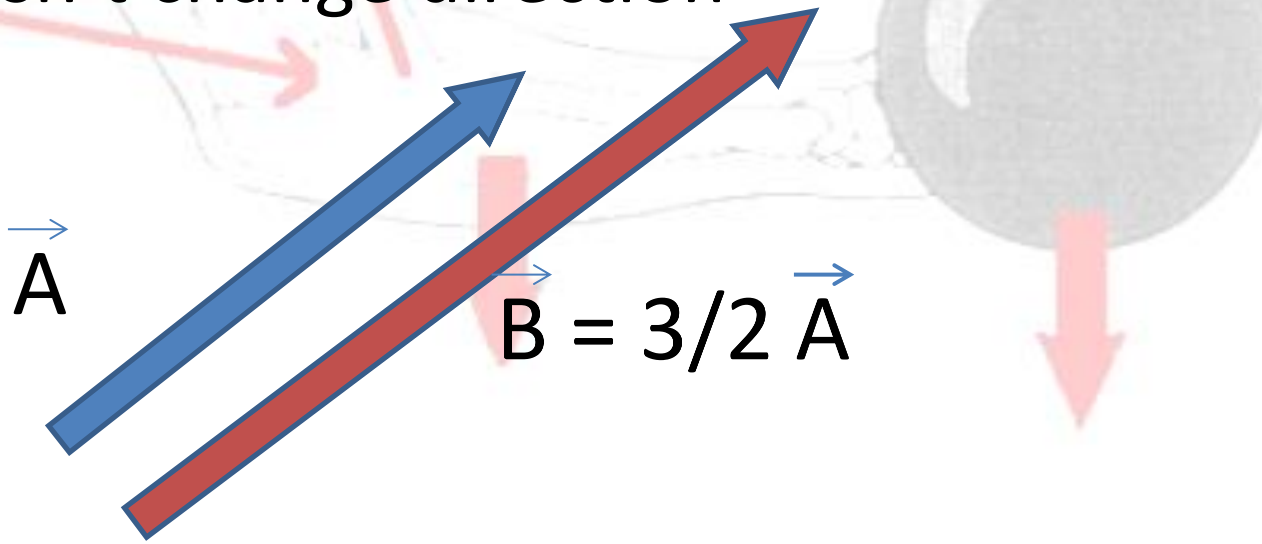
Adding Vectors

•Add two vectors by lacing them heel to toe and seeing where the end result finishes.



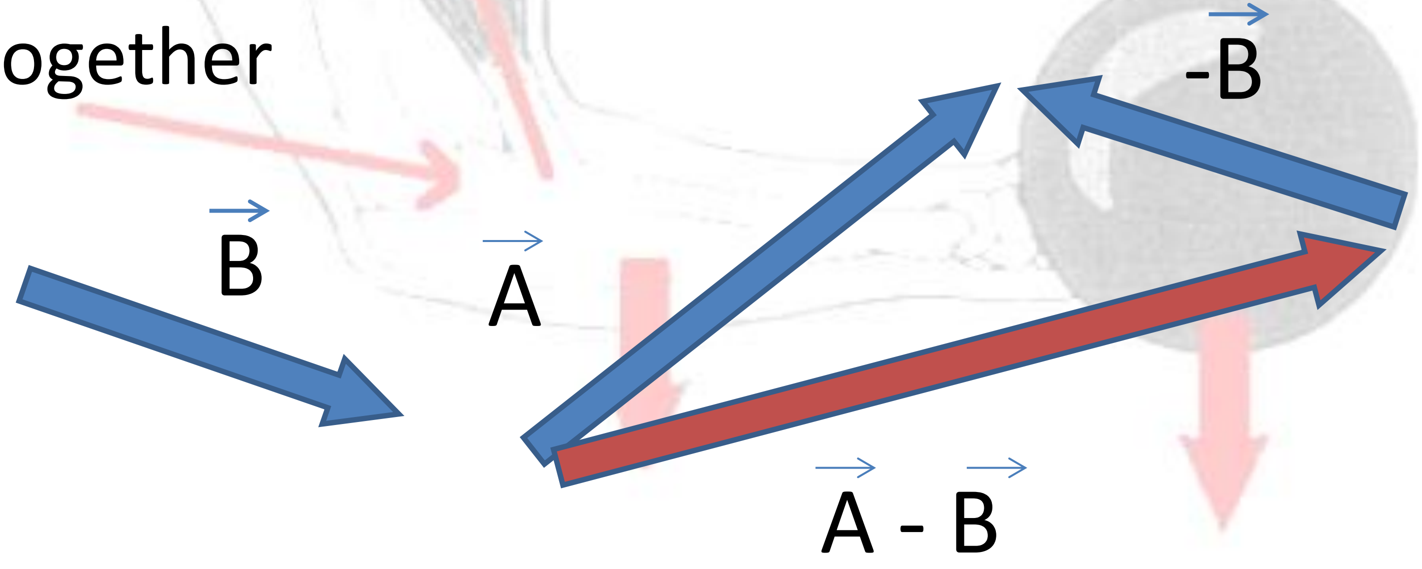
Multiplying a Vector by a Scalar

- Multiplying by a scalar just means multiplying by a number
- Makes vector longer or shorter
- Doesn't change direction



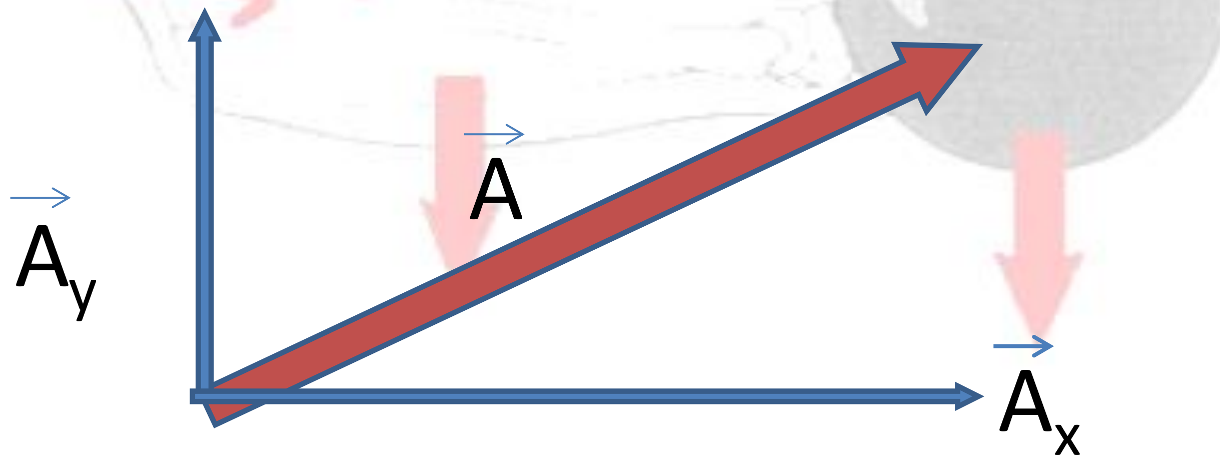
Subtracting Vectors

- Subtracting vectors just means adding a vector multiplied by (-1)
- Turn second vector around, then add together



Components of Vectors

- Easy to express vectors as sum of standard vectors at right angles (orthogonal vectors)
- Gives idea of vector components
  - Have unit vectors, standard step size in each direction



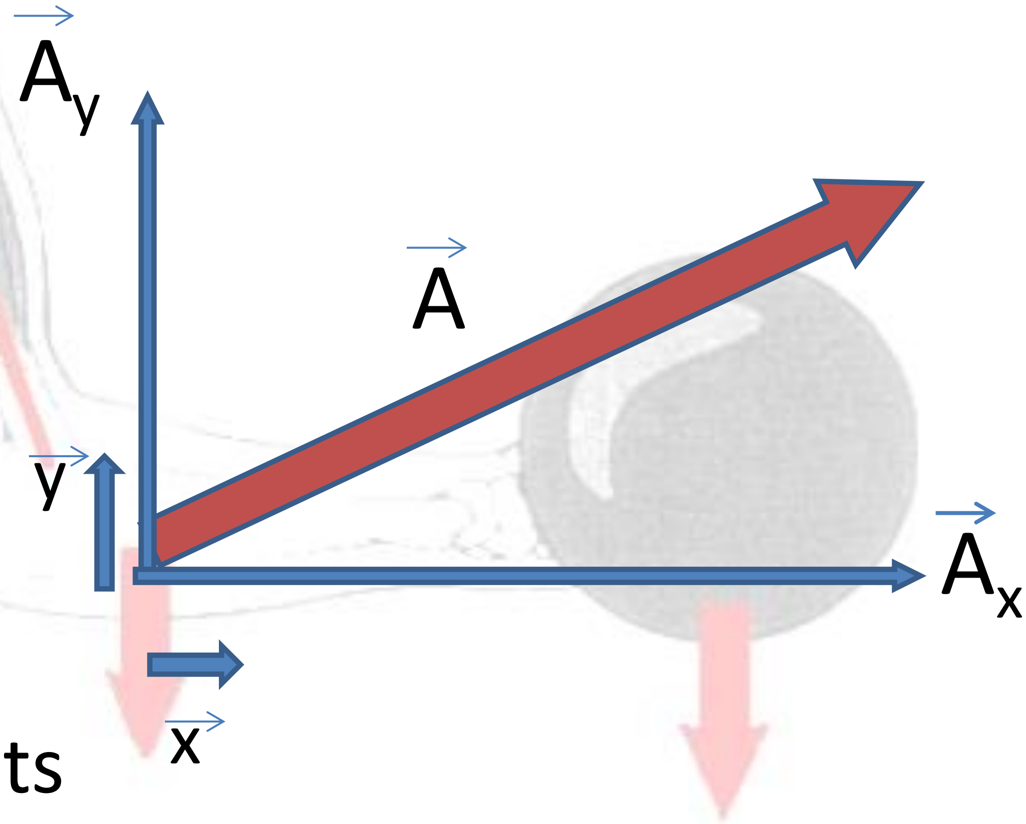
Components of Vectors

$\vec{A} = \vec{A}_x + \vec{A}_y$

$\quad = A_x \vec{x} + A_y \vec{y}$

$\vec{A}_x$ and $\vec{A}_y$ are the component vectors of $\vec{A}$

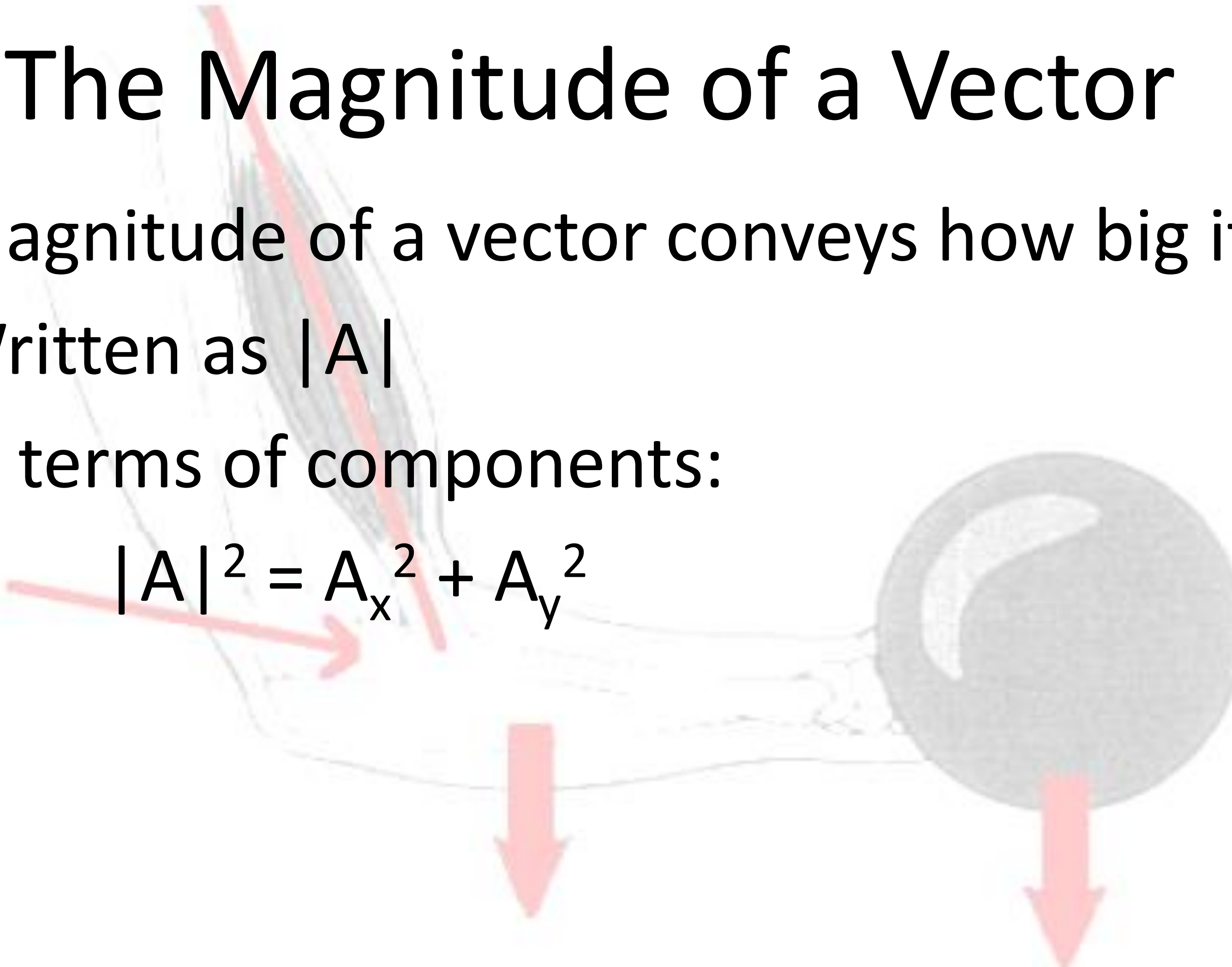
A_x are A_y are the scalar components of A



The Magnitude of a Vector

- Magnitude of a vector conveys how big it is
- Written as |A|
- In terms of components:

$$|A|^2 = A_x^2 + A_y^2$$



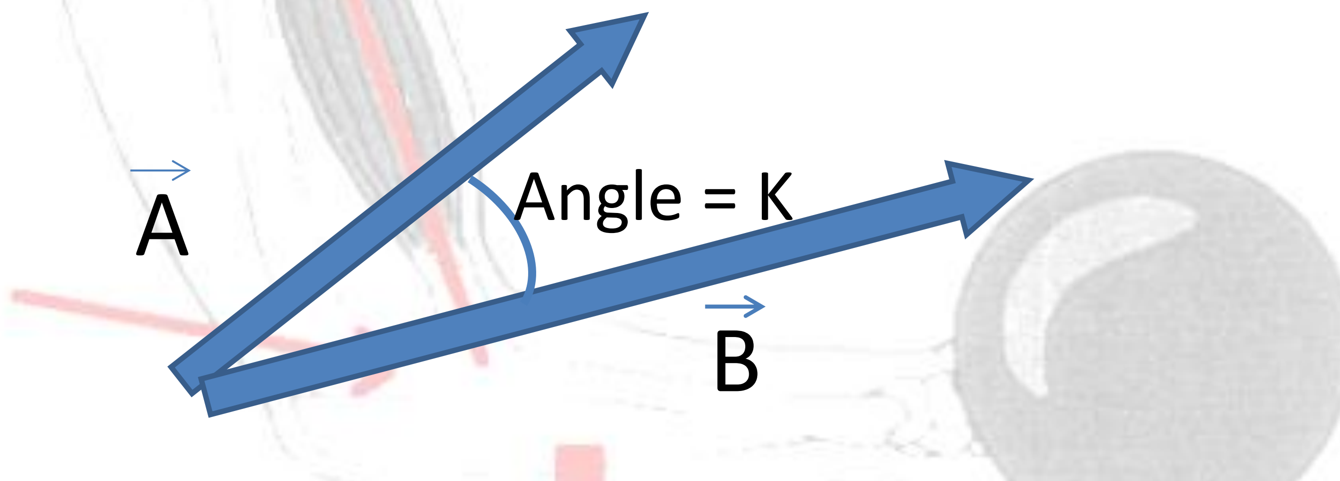
Dot Product of Two Vectors

- Also called scalar product, because outcome is a scalar
- Depends on sizes of vectors but also angle between them
- In terms of components

$$\vec{A}.\vec{B} = A_x\, B_x + A_y\, B_y$$

Dot Product of Two Vectors

- In terms of the graphics:



$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\cos K$$

Cross Product of Vectors

- Also called the vector product
- Multiplies two vectors together to give a third vector
- Resultant vector is at right angles to both vectors multiplied together
    - Needs third dimension
- Size of resultant vectors given by product of two input vectors multiplied by the sine of the angle between them

$$\vec{A} x \vec{B} = \vec{C}$$

$$\left|\vec{C}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin K$$

Cross Product of Vectors

- In terms of components we get

$$\vec{A} x \vec{B} = \vec{C}$$

$$\vec{C} = (A_y B_z - A_z B_y)\vec{x} +$$

$$(A_z B_x - A_x B_z)\vec{y} +$$

$$(A_x B_y - A_y B_x)\vec{z}$$

Cross Product of Vectors

- The direction of the cross product (whether it is up or down from the plane of the two input vectors) is given by the right hand rule.
  - $\vec{A}$ goes along your right thumb
  - $\vec{B}$ goes along your right index finger
  - $\vec{C}$ goes along your right middle finger
- We get that

$$\vec{x} \times \vec{y} = \vec{z}$$

Comparing Dot and Cross Products

Dot Product

- Produces a scalar
- Magnitude given by:

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos K$$

- Dot product is reversible

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

Cross Product

- Produces a vector
- Magnitude given by:

$$|\vec{A} x \vec{B}| = |\vec{A}||\vec{B}|\sin K$$

- Cross product is not reversible

$$\vec{A} x \vec{B} = -\vec{B} x \vec{A}$$

  – It produces a different vector pointing in the opposite direction

Volume of a Box

- Combining dot and cross product gives a handy way to figure out the volume of a box
- Three vectors will define the edges of a box

$$volume = \vec{A}.(\vec{B} x \vec{C})$$