Biomechanics

- Linear motion
- Momentum
- Forces
- Work energy power
- Elastic/Inelastic

Linear Motion

- **Distance travelled** – metres – *s*
- **Initial velocity** – ms^{-1} – v_o
- **Final velocity** – ms^{-1} – v
- **Acceleration** – ms^{-2} – a
- **Time** – seconds – *t*
- These five parameters linked by a set of equations which we'll look at later
- First, we'll see how they behave graphically

Analysis of 100m record

- Usain Bolt, Berlin, 23rd August 2009
- Usain Bolt World Record

Biomechanical analysis

12. IAAF World Championships in Athletics Berlin, 15. - 23.08.2009

berlin 2009

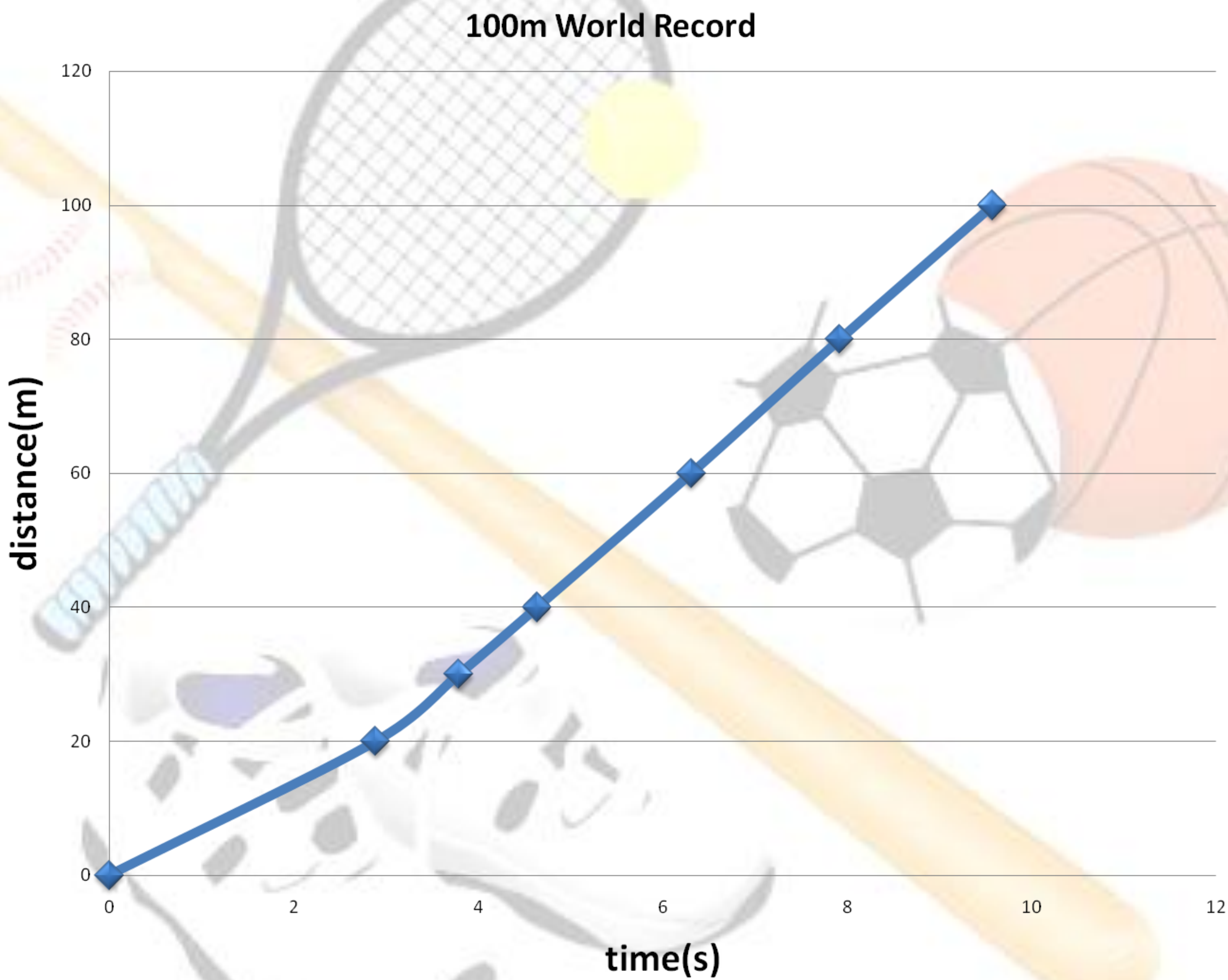
Scientific Research Project

100m Men

Semifinal/Final

		Round	Wind	RT	t_{20m}	t_{40m}	t_{60m}	t_{80m}	t_{100m}	t_{20-40}	t_{40-60}	t_{60-80}	t_{80-100}	t_{30m}	t_{30-60}
Bolt Usain	JAM	Fi	0,9	0,146	2,89	4,64	6,31	7,92	**9,58**	1,75	1,67	1,61	1,66	**3,79**	**2,52**
	JAM	SF 1	0,2	0,135	2,89	4,68	6,41	8,11	**9,89**	1,79	1,73	1,70	1,78	**3,81**	**2,60**
Gay Tyson	USA	Fi	0,9	0,144	2,92	4,70	6,39	8,02	**9,71**	1,78	1,69	1,63	1,69	**3,83**	**2,56**
	USA	SF 2	-0,2	0,143	2,99	4,80	6,54	8,21	**9,93**	1,81	1,74	1,67	1,72	**3,92**	**2,62**
Powell Asafa	JAM	Fi	0,9	0,134	2,91	4,71	6,42	8,10	**9,84**	1,80	1,71	1,68	1,74	**3,83**	**2,59**
	JAM	SF 2	-0,2	0,133	2,92	4,73	6,47	8,17	**9,95**	1,81	1,74	1,70	1,78	**3,85**	**2,62**
Bailey Daniel	ANT	Fi	0,9	0,129	2,92	4,73	6,48	8,18	**9,93**	1,81	1,75	1,70	1,75	**3,85**	**2,63**
	ANT	SF 1	0,2	0,135	2,93	4,74	6,49	8,19	**9,96**	1,81	1,75	1,70	1,77	**3,86**	**2,63**
Thompson Richard	TRI	Fi	0,9	0,119	2,90	4,71	6,45	8,17	**9,93**	1,81	1,74	1,72	1,76	**3,83**	**2,62**
	TRI	SF 2	-0,2	0,132	2,92	4,74	6,51	8,22	**9,98**	1,82	1,77	1,71	1,76	**3,85**	**2,66**
Chambers Dwain	GBR	Fi	0,9	0,123	2,93	4,75	6,50	8,22	**10,00**	1,82	1,75	1,72	1,78	**3,86**	**2,64**
	GBR	SF 2	-0,2	0,182	2,96	4,79	6,55	8,26	**10,04**	1,83	1,76	1,71	1,78	**3,90**	**2,65**
Burns Marc	TRI	Fi	0,9	0,165	2,94	4,76	6,52	8,24	**10,00**	1,82	1,76	1,72	1,76	**3,87**	**2,65**
	TRI	SF 1	0,2	0,159	2,95	4,76	6,52	8,23	**10,01**	1,81	1,76	1,71	1,78	**3,88**	**2,64**
Patton Darvis	USA	Fi	0,9	0,149	2,96	4,85	6,65	8,42	**10,34**	1,89	1,80	1,77	1,92	**3,93**	42,72
	USA	SF 1	0,2	0,152	2,96	4,78	6,51	8,21	**9,98**	1,82	1,73	1,70	1,77	**3,89**	**2,62**

time(s)	distance (m)
0	0
2.89	20
3.79	30
4.64	40
6.31	60
7.92	80
9.58	100



100m World Record
distance(m)
time(s)
0
20
40
60
80
100
120
0
2
4
6
8
10
12

Analysis of Graph

- Slope of line gives his speed
- Curvature of line gives his acceleration

Using the Equations to Solve Problems

- Look at the problem
  - Figure out that it's a kinematic problem
  - Identify all the given quantities in the problem
  - Write them down with their symbols
  - Make sure they are all in S.I. units.
- Write down symbol for unknown quantity.
- Choose correct equation from the list
  - equation which contains the unknown quantity AND for which all the other quantities are known.
- Rearrange the equation so that the unknown quantity is on it's own on left hand side.
- Substitute values for all the known quantities.
- Solve the equation for the unknown quantity.
- Give its units.

Equation 1:

$\underline{v} = \underline{v}_0 + \underline{a}\, t$ (s is missing)

Equation 2:

$\underline{s} = \underline{v}_0\, t + 1/2\, \underline{a}\, t^2$ (v is missing)

Equation 3:

$\underline{v}^2 = \underline{v}_0{}^2 + 2\, \underline{a}\, \underline{s}$ (t is missing)

Equation 4:

$\underline{s} = (\underline{v}_0 + \underline{v})\, t \;/\; 2$ (a is missing)

Problems on Kinematics

1/ If a stone is dropped from the top of a building and takes 5 s to reach the ground, how tall is the building?

- $v = ?$
- $v_o = 0 m/s$
- $a = 9.81 m/s^2$
- $s = ?$
- $t = 5s$

$$v = v_o + at$$

$$s = v_o t + ½at^2$$

$$v^2 = v_o^2 + 2as$$

$$2s = (v + v_o)t$$

- Need an equation that features s along with v_o , a, and t.
- $s = v_o t + ½at^2$
- Equation doesn't need rearranging (s is on left hand side)
- s = 0.5 + ½ 9.81 52 = 0 + 122.6m = 120m
- **height of building is 120m**

- 2/ A ball dropped from the top of a building strikes the ground in 10s. Find the height of the building and the velocity with which the ball hits the ground.
  - **[490.3m, 98.1m/s]**
- 3/ A truck travelling at 20m/s decelerates uniformly to 10m/s over a distance of 100m. Determine the rate of deceleration of the truck and the time required for the truck to travel the 100m.
  - **[-1.5m/s2, 6.7s]**
- 4/ A car moves from rest with a constant acceleration of 5m/s^2. Find the instantaneous speed at the end of 8s, the distance covered in 8s, and the average speed for the 8s interval.
  - **[40m/s, 160m, 20m/s]**
- 5/ A baseball is thrown vertically upward with a velocity of 28m/s. What is the highest point of it’s trajectory?
  - **[40m]**

Dynamics

- Work for this week will focus on:
  - Momentum
  - Forces
  - Work
  - Energy
  - Power

Momentum – intro

- Product of mass and velocity
- Momentum is conserved, always
- Means total momentum before and after a collision is identical
- Momentum can be positive or negative, for things travelling in opposite directions
  - This is because momentum is a vector
- Units for momentum are (mass) x (velocity) or $kgms^{-1}$
- Symbol for momentum is p

Momentum - examples

- Let's look at some examples:
  - Calculate momentum of a 72kg person running at 5 ms^{-1}.
    - $P = mv = 72 \times 5 = 360 kgms^{-1}$
  - Calculate the momentum of a hockey puck (0.16kg) moving at $22ms^{-1}$
    - $P = mv = 0.16 \times 22 = 3.52 kgms^{-1}$
  - Calculate the momentum of a hockey puck (0.16kg) moving at $22ms^{-1}$
    - $P = mv = 0.16 \times 22 = 3.52 kgms^{-1}$

Momentum - conservation

- The reason we care about momentum is because it's conserved
- In any collision we get:
  - (total momentum before) = (total momentum after)
- Powerful way of solving impact problems
  - Don't need information about details of collision to work out key details

Momentum - Conservation

- Some examples:

  – A golf club with a clubhead of 0.35kg strikes a golf ball of mass 0.046kg. The clubhead speed before impact is $42ms^{-1}$, after impact it is $33.2ms^{-1}$. Calculate the launch speed of the golf ball.

$$(\text{total momentum before}) = (\text{total momentum after})$$

$$\Rightarrow m_c v_{c_i} = \mathrm{m}_c v_{c_f} + \mathrm{m}_b v_{b_f}$$

$$\Rightarrow m_b v_{b_f} = m_c v_{c_i} - m_c v_{c_f}$$

$$\Rightarrow m_b v_{b_f} = 0.35 \times 42 - 0.35 \times 33.2$$

$$\Rightarrow m_b v_{b_f} = 14.7 - 11.6 = 3.1$$

$$\Rightarrow v_{b_f} = \frac{3.1}{m_b} = \frac{3.1}{0.046} = 67ms^{-1}$$

Momentum - conservation

– A bowling ball of mass 7kg strikes a bowling pin of mass 1.5kg. The initial speed of the bowling ball is $11ms^{-1}$, after impact it is $6ms^{-1}$. Calculate the speed of the pin after it is hit.

$$(\text{total momentum before}) = (\text{total momentum after})$$

$$\Rightarrow m_b v_{b_i} = \mathrm{m}_b v_{b_f} + \mathrm{m}_p v_{p_f}$$

$$\Rightarrow m_p v_{p_f} = m_b v_{b_i} - m_b v_{b_f}$$

$$\Rightarrow m_p v_{p_f} = 7 \times 11 - 7 \times 6$$

$$\Rightarrow m_p v_{p_f} = 77 - 42 = 35$$

$$\Rightarrow v_{p_f} = \frac{35}{m_b} = \frac{35}{1.5} = 23ms^{-1}$$

Momentum - conservation

– The cue ball in snooker has a mass of 0.4kg. It strikes a stationary red ball of mass 0.38kg at a speed of 2.3ms^{-1}. It recoils at a speed of 0.5ms^{-1}. Calculate the speed of the red ball after the impact.

$$(\text{total momentum before}) = (\text{total momentum after})$$

$$\Rightarrow m_c v_{c_i} = \mathrm{m}_c v_{c_f} + \mathrm{m}_r v_{r_f}$$

$$\Rightarrow m_r v_{r_f} = m_c v_{c_i} - m_c v_{c_f}$$

$$\Rightarrow m_r v_{r_f} = 0.4 \times 2.3 - 0.4 \times (-0.5)$$

$$\Rightarrow m_r v_{r_f} = 0.92 + 0.2 = 1.12$$

$$\Rightarrow v_{r_f} = \frac{1.12}{m_r} = \frac{1.12}{0.38} = 2.95 ms^{-1}$$

- Note how recoil of cue ball is in opposite direction so velocity is negative

Forces

- During the collisions of the previous problems there was a very sharp change in velocity
- This gives very short, very large accelerations
- Like all accelerations, they are produced by forces
- Whenever there is an acceleration there is a force acting

Forces – types of force

- Collisions involve impact forces
  - Typically very large but of short duration
- In previous problems we've met the force of gravity
  - Leads to acceleration of $9.81ms^{-2}$ downwards
- Other forces include friction
  - Walking, driving cars etc
- Pushes and pulls are forces
- Get forces from stretching or compressions springs
  - Restoring forces

Forces – and acceleration

- Forces are connected to accelerations
- Whenever there is an acceleration there is an unbalanced force
- Mathematical link between them given by Newton's Laws
- $F = ma$
- Really defines what a force is

Forces – units

- From F = ma we can figure out the SI unit for force
- Get F to be measured in $kgms^{-2}$
- Give this a special name, the Newton (N)

Forces – gravity

- We've met mass already
- Similar concept is weight
- Weight = mass x gravity
- W = mg
- Weight is a force, measured in Newtons
- Example; weight of a 72kg person is:
  - – W = 72 x 9.81 = 706.32N
- Usually when we measure mass we actually measure weight and figure out the g

Forces – friction1

- Surfaces in contact
- Opposes relative motion
- Can push against an object but it won't move
  - Friction force grows to counteract the push
  - Only when the friction gets to its maximum value will we get movement
  - Usually, oncew something is moving the friction reduces slightly
  - Called statis and dynamic friction

Forces – friction2

- Size of friction force depends on:
  - How rough or smooth the surfaces are
  - Strength of force bonding them together
- Equation for friction is $F_{friction} = \mu mg$
  - Assumes surfaces horizontal
  - Assumes nothing unusual welding surfaces together
- μ is the coeffient of friction
  - just a number
  - is zero for incredibly smooth surfaces
  - is one for extremely rough surfaces

Forces – problems

1/ A box of mass 2 kg rests on a horizontal surface. What is the normal reaction of the surface on the box? ($g = 9.81\ m\ s^{-2}$)

- **[19.62N]**

2/ A wooden box of mass 3 kg is resting on a horizontal floor. The co-efficient of friction between the box and the floor is 0.4. A horizontal force of 10 N is applied to the box. Will the box move? Explain. ($g = 9.81\ m\ s^{-2}$)

- **[no, 12N versus 10N]**

3/ A block of mass 6 kg is sent sliding across a horizontal surface and is observed to decelerate at a rate of 9.2 ms^{-2} because of the presence of friction. Calculate the decelerating force on the block and the coefficient of kinetic friction between the block and the surface. If a 10 kg mass was attached to the top of the block and the experiment repeated would the observed deceleration be greater than, less than, or equal to 9.2 ms^{-2}?

- **[55.2 N, 0.94]**

4/ What force is required to produce an acceleration of 5 ms^{-2} for an object of mass 5 kg? It is observed in a real situation that when this force is applied to a mass of 5 kg the acceleration produced is only 3 ms^{-2} due to friction. Calculate the frictional force and the coefficient of friction.

- **[25 N, 10 N, 0.2]**

5/ A wooden block is sliding across a glass table at 3.7 ms^{-1}. If the coefficient of friction of wood on glass is 0.26 calculate the distance the block will travel before coming to rest.

- **[2.68 m]**

Work

- Work is done when a force moves through a distance
- Ingredients are force and movement
- Equation is work = force x distance
- (technically the force and distance have to be lined up, so there's an angle term as well)
- Units for work are (Newtons) x (metres)
- Call these Joules

Work - Examples

- A 82kg runner accelerates from rest to $4ms^{-1}$ in 15m.
  - Show that the acceleration of the runner is $0.533ms^{-2}$
  - Show that the acceleration force of the runner is 43.73N
  - Show that the work done by the runner is 656J

Energy

- Energy is the ability to do work
- Energy comes in lots of different forms
- During a process, energy can change from one form to another
- But, overall, energy is conserved
- Thermal energy is awkward, can't always convert back to mechanical energies

Energy - Types

- For each type of work there is a corresponding type of energy
- Energy due to motion = Kinetic energy = $\frac{1}{2} mv^2$
- Gravitational potential energy = mgh
- Spring (elastic) potential energy = $\frac{1}{2} kx^2$

Work - Examples

- Work done acceleration a body = ½ mv^2
- Calculate the kinetic energy of a 82kg runner moving at $4ms^{-1}$
  - Use kinetic energy = ½ mv^2
  - =>K.E. = ½ 82 x 4^2
  - =>K.E. = 656J
  - Compare with problem on slide 28

Energy & Food

- Food energy often measured in Calories (sometimes called kcal)
- 1 Calorie = 4184J = 4.184kJ
- Energy content of different foods given below:

food	Energy (kJ/g)	Energy (kcal/g)
Carbohydrates	17.2	4.1
Proteins	17.2	4.1
Fat	38.9	9.3
Potato	3.89	0.93
Eggs	6.82	1.63
Butter	30.1	7.20
Apples	2.43	0.58

Energy & Food-Oxidation of glucose

- Energy can come from oxidation of glucose:

$$C_6H_{12}O_6 + 6O_2 \Rightarrow 6CO_2 + 6H_2O$$

- For every 1g of glucose we get 16kJ of energy
- This requires 0.75L of oxygen
- So get 21kJ per litre of oxygen

Energy & Food-Fat Breakdown

- Energy can come from fats:

  $C_3H_5O_3(OC_4H_7)_3 + 18.5O_2 \Rightarrow 15CO_2 + 13H_2O$

- For every 1g of fat we get 26.9kJ of energy
- This requires 1.37L of oxygen
- So get 19.6kJ per litre of oxygen

Energy & Food-Energy Storage

- Body doesn't use this energy directly, uses ATP to store it
- For example, every time a glucose molecule is burnt it produces 38 molecules of ATP
- Only 44% of energy from glucose is captured by the ATP (rest becomes heat)
- The energy from ATP is released when we require (e.g. for muscle contraction)
  - Hydolysis of ATP to ADP
  - About half of the energy of ATP becomes useful work
  - Overall efficiency of ATP use about 20%, at best

Energy & Food-Oxygen usage

- As we've seen, both glucose and fat produce about 20kJ of energy for every litre of oxygen breathed in
- Easy to measure oxygen intake, multiplying this by 20kJ gives us the *catabolic rate* (rate of production of energy in Watts)
- Table below gives catabolic rate for some activities:

activity	Power (W)	Oxygen consumption (L/min)
Sleeping	83	0.25
Walking	200	0.60
Cycling (25km/hr)	1000	3.0
Tennis	440	1.32
Basketball	800	2.40
Swimming	475	1.43

- Oxygen intake is about 5% of inhaled air

Energy & Food Efficiency

- Efficiency measures the amount of useful work we get divided by the amount of energy we consume
- Depends on type of activity
- Always need about 83W of energy from food (*Basal Metabolic Rate* or *BMR*)
- Using large muscle groups such as in your legs gives efficiencies of ~20%
- Using smaller muscle groups such as upper body gives efficiencies of ~3%

Energy & Food Efficiency

- The amount of energy required to run a certain distance is independent of time taken, pretty much
- But faster speeds require greater powers.
- Empirical relation is:

  energy required = 3.89kJ x (person's mass) x (distance run)
- E.g. running 3km for an 85kg person requires almost 1000kJ
- Same as about 25g of fat (say, 3 slices of bread)

Energy - mechanical

- Special class of “flexible” energies
  - Kinetic
  - Potential
    - Gravitational
    - Elastic
- When these are conserved process is called ***Elastic***
- Otherwise process is called ***Inelastic***

Energy - Example

- Let's re-examine the problem from slide 16
  - A golf club with a clubhead of 0.35kg strikes a golf ball of mass 0.046kg. The clubhead speed before impact is $42ms^{-1}$, after impact it is $33.2ms^{-1}$. Using momentum conservation we calculated the launch speed of the golf ball to be $67ms^{-1}$.
  - Is this an elastic collision?
  - KE before collision is $KE_{club} + KE_{ball} = 308.7J$
  - KE after collision is $KE_{club} + KE_{ball} = 296.1J$
  - Inelastic collision
  - Some energy has been lost to heat, sound, vibrations, etc

Energy – Inelastic Collisions

- Need some way to express amount of inelasticity in collision
- Define **Coefficient of Restitution** (COR)

$$COR = \frac{v_{A_{final}} - v_{B_{final}}}{v_{A_{initial}} - v_{B_{initial}}}$$

Energy – Inelastic Collisions

- What is the COR for the golf ball from slide 33?

$$COR = \left| \frac{v_{A_{final}} - v_{B_{final}}}{v_{A_{initial}} - v_{B_{initial}}} \right|$$

$$\Rightarrow COR = \left| \frac{67 - 33.2}{42 - 0} \right|$$

$$\Rightarrow COR = 0.804$$

Energy – Inelastic Collisions

- If collision is perfectly elastic then the COR is 1.0
- If collision is perfectly inelastic then the COR is 0
- For bouncing balls, can use $v^2 = v_o^2 + 2as$ to get another version of the COR equation:

$$COR = \sqrt{\frac{\text{bounce height}}{\text{drop height}}}$$

- Permissible COR's regulated by sports bodies

Power

- Rate of doing work
- Power = (work done) / (time taken)
- Units are Joules per second, or just **Watts**
- For muscles, maximum power output comes at about half their maximum speed of contraction
- Maximum power output also depends on duration of activity
- 3 to 6 Watts for each kg of body mass for one hour
- 25 Watts per kg possible for short spells

Power - example

- A sprinter of mass 82kg produces 2050W of power for 1.8s to accelerate his body from rest. Calculate the speed of the sprinter after 1.8s.
  - Total energy produced is Power x time = 3690J
  - Kinetic energy = ½ mv^2 = 3690
  - Therefore final speed = v = 9.49ms^{-1}

Power - example

- A 68kg cyclist is climbing a 1 in 4 hill. If the cyclist is producing 204W of power, calculate his speed (neglect the weight of the bicycle, in a 1 in 4 hill for every metre you climb you move forward 4.12m)
  - Power used to climb hill, gravitational energy
  - power x time = mgh
  - Therefore power = mgh / t
  - Therefore rate of climb = h/t = P/mg = 204/(68x9.81)
  - h/t = $0.306ms^{-1}$
  - this means forward speed = 0.306 x 4.12 = $1.26ms^{-1}$

Biomechanics Summary

- Distance / speed / acceleration
- Distance/time graphs
- Kinematic equations
- Momentum, conservation
- Forces
  - Gravity
  - Friction
- Work
- Energy
- Elastic / inelastic
- Coefficient of Restitution
- power

Biomechanics Summary

- $v = s/t$
- $\underline{v} = \underline{v_0} + \underline{a}\, t$
- $\underline{s} = \underline{v_0}\, t + 1/2\, \underline{a}\, t^2$
- $\underline{v}^2 = \underline{v_0}^{\,2} + 2\, \underline{a}\, \underline{s}$
- $\underline{s} = (\underline{v_0} + \underline{v})\, t \;/\; 2$
- $p = mv$
- $P_{before} = p_{after}$
- $F=ma$

- $W = mg$
- $F_{fric} = \mu mg$
- $Work = F\, s$
- $KE = ½\, mv^2$
- $GPE = mgh$
- $EPE = ½\, kx^2$
- $COR = (bounce/drop)^{½}$
- $P = energy\ /\ time$